Target Independence of the 'Proton Spin' Effect

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Recent work by the author in collaboration with S. Narison and G. Veneziano on the EMC-SMC-SLAC 'proton spin' effect is reviewed. This uses a novel approach to deep inelastic scattering in which the matrix elements arising from the OPE are factorised into composite operator propagators and proper vertices. For polarised μp scattering, the composite operator propagator is equated to the square root of the first moment of the QCD topological susceptibility, $\sqrt{\chi'(0)}$. We evaluate $\chi'(0)$ using QCD spectral sum rules and find a significant suppression relative to its OZI expectation. This is identified as the source of the violation of the Ellis-Jaffe sum rule for the first moment of the polarised proton structure function g_1^p. Our predictions, $\int_0^1 dx g_1^p(x; Q^2 = 10GeV^2) = 0.143 \pm 0.005$ and $\Delta\Sigma = 0.353 \pm 0.052$, are in excellent agreement with the new SMC data. This supports our earlier conjecture that the suppression in the flavour singlet component of the first moment of g_1^p is a target-independent feature of QCD related to the $U(1)$ anomaly and is not a special property of the proton structure.

1. Introduction

The discovery of an anomalous suppression in the first moment of the polarised proton structure function g_1^p by the EMC collaboration[1] in 1988 stimulated a period of intense theoretical and experimental activity in the QCD community. Recently, the existence of this so-called 'proton spin problem' has been confirmed by new experiments at CERN[2] and SLAC[3], although the numerical results have been substantially revised.

In this contribution, I will review some recent work[4] with S. Narison and G. Veneziano which we believe gives a theoretically convincing and quantitative resolution of the problem. In our picture[5], the 'proton spin' effect is seen as a natural addition to the class of OZI-violating $U(1)$ phenomena characteristic of QCD in the flavour singlet pseudoscalar (or pseudovector) channel. Furthermore, the violation of the Ellis-Jaffe sum rule, due to the suppression relative to the OZI expectation of the singlet form factor $G_A^{(0)}$ (usually denoted by $\Delta\Sigma$), is recognised as a generic, target-independent feature of QCD related to the axial $U(1)$ anomaly and not as a special property of the proton structure. In fact, it reflects an anomalous suppression of the first moment of the QCD topological susceptibility, $\chi'(0)$.

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2. The first moment sum rule for g_1^p

The sum rule for the first moment of the polarised proton structure function g_1^p reads:

$$\begin{aligned}\Gamma_1^p(Q^2) \quad &\equiv \int_0^1 dx g_1^p(x;Q^2) \\ &= \frac{1}{6}\Big[\Big(G_A^{(3)}(0) + \frac{1}{\sqrt{3}} G_A^{(8)}(0)\Big)\Big(1 - \frac{\alpha_s}{\pi}\Big) \\ &\qquad + \frac{2}{3} G_A^{(0)}(0;Q^2)\Big(1 - \frac{1}{3}\frac{\alpha_s}{\pi}\Big)\Big] \end{aligned} \quad (1)$$

where the $G_A^{(a)}(k^2)$ are form factors in the proton matrix elements of the axial current:

$$\langle P|J^a_{\mu 5R}(k)|P\rangle = G_A^{(a)} \bar{u}\gamma_\mu\gamma_5 u + G_P^{(a)} k_\mu \bar{u}\gamma_5 u \quad (2)$$

and a is an $SU(3)$ flavour index. Here, we just display the perturbative corrections to $O\big(\alpha_s(Q^2)\big)$. For further terms, see [4]. Since our results depend smoothly on the quark masses in the chiral limit, we set the light quark masses to zero.

The interpretation of these form factors in the naive parton model is, in standard notation:

$$\begin{aligned} G_A^{(3)}(0) \quad &= \frac{1}{2}(\Delta u - \Delta d) \\ G_A^{(8)}(0) \quad &= \frac{1}{2\sqrt{3}}(\Delta u + \Delta d - 2\Delta s) \end{aligned}$$

$$G_A^{(0)}(0) \equiv \Delta\Sigma = \Delta u + \Delta d + \Delta s \tag{3}$$

The axial current occurs here since it is the lowest twist, lowest spin, odd-parity operator in the OPE of two electromagnetic currents (see sect.4). The suffix R emphasises that the current is the renormalised composite operator. Under renormalisation, the gluon topological density Q_R and the divergence of the flavour singlet axial current $J^0_{\mu 5R}$ mix as follows:

$$\begin{aligned} J^0_{\mu 5R} &= Z J^0_{\mu 5B} \\ Q_R &= Q_B - \frac{1}{2N_F}(1-Z)\partial^\mu J^0_{\mu 5B} \end{aligned} \tag{4}$$

where $J^0_{\mu 5B} = \sum \bar{q}\gamma_\mu\gamma_5 q$ and $Q_B = \frac{\alpha_s}{8\pi}\mathrm{tr}G^{\mu\nu}\tilde{G}_{\mu\nu}$ and we have quoted the formulae for N_F flavours. The mixing is such that the combination occurring in the axial anomaly Ward identities, e.g.

$$\langle 0|\Big(\partial^\mu J^0_{\mu 5R} - 2N_F Q_R\Big)\mathcal{O}|0\rangle + \langle 0|\delta_5\mathcal{O}|0\rangle = 0 \tag{5}$$

is not renormalised.

Since $J^0_{\mu 5R}$ is renormalised, its matrix elements satisfy renormalisation group equations with an anomalous dimension γ, so that in particular $G_A^{(0)}$ depends on the RG scale, set to Q^2 in eq.(1).

We emphasise that $G_A^{(0)}$ does *not*, as was initially supposed, measure the spin of the quark constituents of the proton. The RG non-invariance of $J^0_{\mu 5R}$, which is a consequence of the anomaly, means that it is *not* a conserved current. Only in the idealised case of a free Dirac field do the matrix elements of $J^0_{\mu 5}$ and spin coincide.

To compare the sum rule with experiment, we use the following standard results:

$$G_A^{(3)}(0) = \frac{1}{2}\big(F+D\big), \quad G_A^{(8)}(0) = \frac{1}{2\sqrt{3}}\big(3F-D\big) \tag{6}$$

where $F+D = 1.257 \pm 0.008$ and $F/D = 0.575 \pm 0.016$ as fitted from hyperon and beta decays. We also take $\alpha_s(m_\tau) = 0.347 \pm 0.030$ from tau decay data.

The Ellis-Jaffe sum rule is obtained by assuming that, in parton language, the strange quark polarisation in the proton vanishes, i.e. $\Delta s = 0$ in eq.(3). This is equivalent to the OZI prediction

$$G_A^{(0)}(0)\big|_{\mathrm{OZI}} = 2\sqrt{3}G_A^{(8)}(0) = 0.579 \pm 0.021 \tag{7}$$

and corresponds to

$$\Gamma_1^p(Q^2 = 10GeV^2) = 0.170 \pm 0.003 \tag{8}$$

In contrast, our result

$$G_A^{(0)}(0)\big|_{Q^2=10GeV^2} = 0.353 \pm 0.052 \tag{9}$$

leads to

$$\Gamma_1^p(Q^2 = 10GeV^2) = 0.143 \pm 0.005 \tag{10}$$

We can also compare with the lowest order expectation in the Skyrme model[6], according to which the proton decouples from the flavour singlet pseudoscalar channel ($\Gamma_{\Phi_{5R}P\bar{P}} = 0$ in the language of sect.4) so that $G_A^{(0)}(0) = 0$ and $\Gamma_1^p(Q^2 = 10GeV^2) = 0.107 \pm 0.002$

Figure 1. The recent data for Γ_1^p from the SMC collaboration. The data points show $\int_{x_{\min}}^1 dx g_1^p(x)$ at $Q^2 = 10GeV^2$ plotted against $x_{\min}$ and converge to the SMC value of $\Gamma_1^p(Q^2 = 10GeV^2) = 0.136 \pm 0.011 \pm 0.011$. Notice that the 'world average' quoted by SMC is a little higher (see eq.(13)). Also shown are the theoretical predictions of the Ellis-Jaffe sum rule (EJ), the Skyrme model (S) and our own prediction (NSV).

These theoretical expectations are compared with the experimental data in Fig.1. The original EMC result was[1]

$$\Gamma_1^p(Q^2 = 11GeV^2) = 0.126 \pm 0.010 \pm 0.015 \tag{11}$$

which, with the above values for F/D and α_s, allows the following value of $G_A^{(0)}$ to be extracted:

$$G_A^{(0)}\big|_{Q^2=11GeV^2} = 0.19 \pm 0.17 \tag{12}$$

Clearly, this value is barely consistent with our prediction. It is therefore extremely gratifying that the improved analysis by the SMC collaboration quoted in [2] now gives the new 'world average'

$$\Gamma_1^p(Q^2 = 10GeV^2) = 0.145 \pm 0.008 \pm 0.011 \tag{13}$$

from which we deduce

$$G_A^{(0)}\big|_{Q^2=10GeV^2} = 0.37 \pm 0.07 \pm 0.10 \tag{14}$$

Notwithstanding the large experimental errors, the agreement between the new data and our prediction is excellent.

3. The Composite Operator/Proper Vertex Method for DIS

The essential features of our method are easily described for a general deep inelastic scattering process. The hadronic part of the scattering amplitude is given by the imaginary part of the two-current matrix element $\langle N|J_\mu(q)J_\nu(-q)|N\rangle$. The OPE is used to expand the large Q^2 limit of the product of currents as a sum of Wilson coefficients $C_i(Q^2)$ times renormalised composite operators $\mathcal{O}_i$ as follows (suppressing Lorentz indices),

$$J(q)J(-q) \underset{Q^2\to\infty}{\sim} \sum_i C_i(Q^2)\mathcal{O}_i(0) \tag{15}$$

The dominant contributions to the amplitude arise from the operators $\mathcal{O}_i$ of lowest twist. Within this set of lowest twist operators, those of spin n contribute to the nth moment of the structure functions, i.e.

$$\int_0^1 dx x^{n-1} F(x;Q^2) = \sum_i C_i^n(Q^2)\langle N|\mathcal{O}_i^n(0)|N\rangle \tag{16}$$

The Wilson coefficients are calculable in QCD perturbation theory, so the problem reduces to evaluating the target matrix elements of the corresponding operators. We now introduce appropriately defined proper vertices $\Gamma_{\tilde{\mathcal{O}}N\bar{N}}$, which are chosen to be 1PI with respect to a physically motivated basis set $\tilde{\mathcal{O}}_j$ of renormalised composite operators. The matrix elements are then decomposed into products of these vertices with zero-momentum composite operator propagators as follows,

$$\langle N|\mathcal{O}_i(0)|N\rangle = \sum_j \langle 0|\mathcal{O}_i(0)\tilde{\mathcal{O}}_j(0)|0\rangle\ \Gamma_{\tilde{\mathcal{O}}_j N\bar{N}} \tag{17}$$

Despite being non-perturbative, we can frequently evaluate the composite operator propagators using a combination of exact Ward identities and dynamical approximations.

All this is illustrated in [4]. In essence, what we have done is to split the whole amplitude into the product of a 'hot QCD' (high momentum) part described by QCD perturbation theory, a 'cold QCD' part described by a (non-perturbative, zero-momentum) composite operator propagator and finally a target-dependent proper vertex. The generic expression for a structure function sum rule is then:

$$\begin{aligned}&\int_0^1 dx x^{n-1} F(x;Q^2)\\ &= \sum_i \sum_j C_i^n(Q^2)\ \langle 0|O_i(0)\tilde{\mathcal{O}}_j(0)|0\rangle\ \Gamma_{\tilde{\mathcal{O}}_j N\bar{N}}\end{aligned} \tag{18}$$

All the target dependence is contained in the vertex function $\Gamma_{\tilde{\mathcal{O}}N\bar{N}}$. However, these are not unique – they depend on the choice of the basis $\tilde{\mathcal{O}}_j$ of composite operators. This choice is made on physical grounds based on the relevant degrees of freedom, the aim being to parametrise the amplitude in terms of a minimal, but sufficient, set of vertex functions. (These play the rôle of the non-perturbative parton distributions in the usual treatment). A good choice can often lead to an almost direct correspondence between the proper vertices and physical couplings such as, e.g., the pion-nucleon coupling $g_{\pi NN}$. In particular, the proper vertices should be chosen whenever possible to be RG invariant.

It is important to realise that the decomposition (17) is an *exact* expression, independent of the choice of the set of operators $\tilde{\mathcal{O}}_j$. A different choice of basis set merely changes the definition of the proper vertices. In particular, it is *not* to be understood that the set $\tilde{\mathcal{O}}_j$ is in any sense a

complete set and that choosing a finite number of operators (such as the pseudoscalars Φ_{5R} and Q_R in sect.4) represents an approximation.

4. The g_1^p Sum Rule and the Topological Susceptibility

We now apply this method to the sum rule for g_1^p. The relevant OPE is:

$$J_\mu(q)J_\nu(-q) \underset{Q^2\to\infty}{\sim} 2\sum_{a=0,3,8} \epsilon_{\mu\nu\alpha}{}^{\beta}\frac{q^\alpha}{Q^2}C^a(Q^2)J^a_{\beta 5R} \quad (19)$$

Assuming the absence of a massless pseudoscalar (Goldstone) boson in the $U(1)$ channel, we find

$$\begin{aligned} G_A^{(0)}(0;Q^2)\bar{u}\gamma_5 u &= \frac{1}{2M}\langle P|\partial^\mu J^0_{\mu 5R}|P\rangle \\ &= \frac{1}{2M}2N_F\langle P|Q_R(0)|P\rangle \quad (20) \end{aligned}$$

where we have used the anomalous chiral Ward identity to re-express $G_A^{(0)}(0)$ as the forward matrix element of the renormalised gluon topological density Q_R. M is the proton mass.

We now choose the composite operator basis $\tilde{\mathcal{O}}_j$ to be the set of renormalised flavour singlet pseudoscalar operators Q_R and Φ_{5R}, where, up to a subtle but crucial normalisation factor (see [4] and [5] for an explanation), the corresponding bare operator is simply $i\sum\bar{q}\gamma_5 q$. We may then write (c.f. eq.(17)):

$$\begin{aligned} &\langle P|Q_R(0)|P\rangle \\ &= \langle 0|Q_R Q_R|0\rangle\Gamma_{Q_R P\bar{P}} + \langle 0|Q_R\Phi_{5R}|0\rangle\Gamma_{\Phi_{5R}P\bar{P}} \quad (21) \end{aligned}$$

where the composite operator propagators are at zero momentum and the proper vertices are 1PI with respect to Q_R and Φ_{5R} only.

The composite operator propagator in the first term in eq.(21) is the zero-momentum limit of an important quantity in QCD known as the topological susceptibility $\chi(k^2)$, viz.

$$\chi(k^2) = \int dx e^{ik.x} i\langle 0|T^* Q_R(x)\ Q_R(0)|0\rangle \quad (22)$$

Moreover, it can be shown exactly using chiral Ward identities[4] that the propagator $\langle 0|Q_R\ \Phi_{5R}|0\rangle$ at zero momentum is simply the square root of the first moment of the topological susceptibility. We therefore find:

$$\langle P|Q_R(0)|P\rangle = \chi(0)\Gamma_{Q_R P\bar{P}} + \sqrt{\chi'(0)}\Gamma_{\Phi_{5R}P\bar{P}} \quad (23)$$

The chiral Ward identities also show that for QCD with massless quarks, $\chi(0)$ actually vanishes. This is in contrast to pure Yang-Mills theory, where $\chi(0)$ is non-zero. We therefore arrive at our basic result[5]:

$$G_A^{(0)}(0;Q^2)\bar{u}\gamma_5 u = \frac{1}{2M}2N_F\sqrt{\chi'(0;Q^2)}\Gamma_{\Phi_{5R}P\bar{P}} \quad (24)$$

The quantity $\sqrt{\chi'(0)}$ is not RG invariant and scales with the anomalous dimension γ. On the other hand, the proper vertex has been chosen specifically so as to be RG invariant. The renormalisation group properties of this decomposition are crucial to our resolution of the 'proton spin problem'.

Our proposal is that we should expect the source of OZI violations to lie in RG non-invariant terms, i.e. in $\chi'(0)$. The reasoning is as follows. In the absence of the $U(1)$ anomaly, the OZI rule would be an exact property of QCD. So the OZI violation is a consequence of the anomaly. But it is the existence of the anomaly that is responsible for the non-conservation and hence non-trivial renormalisation of the axial current $J^0_{\mu 5R}$. We therefore expect to find OZI violations in quantities sensitive to the anomaly, which we identify through their RG dependence on the anomalous dimension γ. This seems reasonable since, if the OZI rule were to be good for such quantities, it would mean approximating a RG non-invariant, scale-dependent quantity by a scale-independent one.

Notice that we are *not* saying that the OZI violation is due to a large (non-perturbative) scaling effect dependent on γ. We are simply using the dependence on the anomalous dimension to identify those quantities most likely to display significant differences from their OZI approximations.

If this proposal is correct, we expect $\sqrt{\chi'(0)}$ to be significantly suppressed relative to its OZI approximation of $(1/\sqrt{6})f_\pi$. The proper vertex $\Gamma_{\Phi_{5R}P\bar{P}}$ would behave exactly as expected according to the OZI rule. That is, the Ellis-Jaffe violating suppression of the first moment of g_1^p observed by EMC would *not* be a special property of the proton at all, but would simply be due to an anomalously small value of the first moment of the QCD topological susceptibility $\chi'(0)$.

This is our conjectured resolution[5] of the 'proton spin problem'. It is further supported by a number of experimental results in the pseudoscalar $U(1)$ channel, notably in $\eta' \to \gamma\gamma$ decays. See [5] for further discussion.

Putting all this together, we conjecture the following expression for the singlet form factor:

$$G_A^{(0)}(0;Q^2) = 2\sqrt{3}G_A^{(8)}(0)\ \frac{\sqrt{\chi'(0;Q^2)}}{(f_\pi/\sqrt{6})} \tag{25}$$

5. $\chi'(0)$ from QCD Spectral Sum Rules

The remaining task is to find a non-perturbative estimate of the first moment of the topological susceptibility, $\chi'(0)$. This is a fundamental quantity likely to reappear in many applications of QCD, and an evaluation from first principles represents a strong challenge to lattice gauge theory. Of course, for a meaningful result it is necessary to work beyond the quenched approximation, close to the chiral limit.

Instead, we estimate $\chi'(0)$ using QCD spectral sum rules. A full description of the calculation is given in [4] and here we only quote the result. We have evaluated $\chi'(0)$ using subtracted dispersion relations with the Laplace sum rule method, finding good stability, and have confirmed the result using the finite energy sum rule technique. The spectral function is saturated with the single lightest pseudoscalar state, the η'. We find

$$\sqrt{\chi'(0)}\Big|_{Q^2=10GeV^2} = 23.2 \pm 2.4 \text{ MeV} , \tag{26}$$

a suppression of approx. 0.6 relative to the OZI value $f_\pi/\sqrt{6}$. Substituting this into eq.(25) finally gives our result (9) for $G_A^{(0)}$.

The essential input parameter in the spectral sum rules is the η' mass. (A quite different result is found for $\chi'(0)$ in pure Yang-Mills theory, saturating the spectral function with a pseudoscalar glueball.) In essence, the sum rules allow us to determine the relevant mass scale for OZI breaking in the pseudovector channel using as input the known OZI-violating η' mass from the pseudoscalar channel. The link is the $U(1)$ Goldberger-Treiman relation[5] which underlies our approach.

6. Conclusion: Not Spin, Not the Proton, Not a Problem!

Our conclusions are simply stated. $\Delta\Sigma$ does *not* measure spin. Its suppression relative to the OZI (Ellis-Jaffe) value is due to an anomalously small value of $\chi'(0)$ in QCD and is *not* a special property of the proton. The violation of the Ellis-Jaffe sum rule is *not* a problem in QCD – the flavour singlet pseudovector channel is precisely where we should expect to find large OZI violations and, using spectral sum rules, we have given a successful quantitative prediction of $\Delta\Sigma$ and Γ_1^p.

The g_1^p sum rule does, however, present some problems for QCD-inspired models of the proton. The Skyrme model would have to be significantly extended to incorporate the $O(1/N_C)$ effects characteristic of the $U(1)$ anomaly. In the parton model, the effect can be incorporated (though not as yet predicted) by modifying the constituent quark expression (3) for $\Delta\Sigma$ to include a polarised gluon density $\Delta\Gamma$ with the necessary renormalisation group behaviour. For further details of this approach, see [7].

Finally, it would be interesting to test our proposal of target independence directly by experiment. This should be possible in semi-inclusive processes in which a pion or D meson is detected.

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